

The University of Chicago

- I. THE PROBLEM OF STATISTICAL DEMAND
TECHNIQUES FOR PRODUCERS' GOODS:
AN APPLICATION TO STEEL
- II. THE STATISTICAL LAW OF DEMAND FOR
A PRODUCER'S GOOD AS ILLUSTRATED
BY THE DEMAND FOR STEEL

A PART OF A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE
SOCIAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

1933

By

ROSWELL HARTSON WHITMAN

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THE PROBLEM OF STATISTICAL DEMAND TECH- NIQUES FOR PRODUCERS' GOODS: AN APPLICATION TO STEEL

ECONOMIC science is no longer principally concerned with the creation of an equilibrium price theory, but instead is now chiefly engaged in the study of those complications in the price system which are not entirely "explained" by such an equilibrium price theory. This new emphasis attempts, by a series of further approximations to the equilibrium system already worked out, to bring within the science a wider range of phenomena, and to develop a theoretical description of the price system more accurate in detail and more useful, particularly for the analysis of the non-equilibrium tendencies of the system.

A number of these further approximations deal with the instabilities which are largely due to the money and credit system, others with the complications arising from the imperfections of competition. A third variety of these approximations, however, of which this paper is an example, attempts to study the unstable price and quantity behavior which may arise in the market for a single commodity. The problem for such analysis is not that of the ultimate distribution of resources, which is the primary problem of equilibrium theory, but rather of the behavior of market demand, and in the present case that of the highly unstable market demand for the commodity steel.

I

Before the problem can be attacked statistically, it must first be determined whether the theory of demand developed for an equilibrium situation will be an adequate basis for the study of the steel market. This demand theory, it should be noted, was extremely useful to the statistical study of the demand for agricultural commodities. Moore, it is true, contended that the Marshallian statement of demand was inadequate, but it is now generally recognized that the Marshallian statement, together with the refinements developed by the Lausanne school, have been fundamental to most of the work done. First of all, the idea of a curve sloping downward and to the right was accepted, and not only was the concept of *ceteris paribus* fairly universal, but the Marshallian statement of the content of the *ceteris paribus* was practically undisputed. Walras, moreover, had already advanced the idea of placing certain of the "other things" in the demand equation itself. The main line of advance in the field came through the belief that the other things could be accounted for, and in the discovery of statistical techniques for accomplishing this. Only slight change in theory was necessary for the statistical study of the demand for agricultural commodities.

While the Marshallian demand concept was adequate for work in the field of agricultural commodities, it was too simple and lacked the necessary flexibility for handling an analysis of a market for a producers' good characterized by a highly unstable market demand. The current market pricing process for these goods, in fact, takes place in a market practically excluded from consideration in either the Marshallian or the Lausanne pricing analysis. Marshall, for example, states that: "A rise in price tends to check consumption; but if the price rise is expected to continue, it will probably, as already noted, lead dealers to increase their stocks."¹ A dealers' market, Marshall therefore believes, will not have the same type of demand conditions as will a consumers' market. It is this dealers' market which *is never given a thorough theoretical analysis*. Marshall's own analysis,

¹ Alfred Marshall, *Principles of Economics* (8th ed.; London, 1920), Book III, chap. iv, 112.

fitting a consumers' market, is inadequate for analyzing a market for a producers' good, since the latter is necessarily a dealers' market.

The neo-classical school never went much farther into this problem. Taussig, it is true, recognized in his article, "Is the Market Price Determinate?"² that there was such a problem. According to his analysis, a penumbra exists in which the simplified statement of demand does not work. This was an admission that there was little current theory to help a student of the concrete nature of demand curves for producers' goods. The Lausanne school, being primarily interested in the problem of general equilibrium rather than that of market equilibrium for a single commodity and in a brief period of time, likewise never went far into the problem.

The development of a reasonable theory for the behavior of the demand for a producers' good is, to a considerable extent, the result of the work of Evans and Roos.³ These two men indicate clearly that for a commodity whose monthly, or even yearly, sales do not offer either a measure of consumption for the same time period or full use of an "equilibrium" plant capacity, the quantity-taken, in the sense of sales, is affected by other price phenomena than the relation of the price of the commodity to other prices in general (the "price" of the usual theoretical analysis). Among these other phenomena Evans and Roos emphasize particularly the direction and rapidity (the rate of change) at which the price in question is moving. They also consider that the relation of present prices to prices in the past will determine in part the pres-

² F. W. Taussig, "Is the Market Price Determinate?" *Quarterly Journal of Economics*, XXXV (1931), 394-411.

³ G. C. Evans, "The Dynamics of Monopoly," *American Mathematical Monthly*, XXXI, No. 2 (February, 1924), 27.

———, *Mathematical Introduction to Economics* (New York: McGraw-Hill Book Co., 1930), p. 143.

———, "A Simple Theory of Economic Crises," *Journal of the American Statistical Association*, XXVI (1931), supplement, 61-72.

C. F. Roos, "Dynamical Theory of Economics," *Journal of Political Economy*, XXXV (1929), 632-56.

———, "A Mathematical Theory of Price Fluctuations," *Journal of Political Economy*, XXXVIII (1930), 513-22.

ent sales. Evans has stated the importance of price movements on quantity-taken as follows:

It [the effect of price change on quantity-taken] is related to the familiar symptom of "prosperity" that the demand for a commodity is greater when the price is increasing than when it is decreasing. An exaggerated instance of this law of demand, perhaps due to an undue generalization from the past crisis, is what the wholesale lumber dealers tell us is characteristic of their sales, namely, when prices are going up the demand is insatiable, but when prices go down it is nil until the price movement stops.⁴

Steel obviously has the same type of demand behavior as lumber. While the production of steel has a relatively fixed upper limit, it can rapidly be altered below this limit, and this, together with the feasibility of booking orders in excess of immediate capacity, makes possible immediate and pronounced shifts in quantity-taken. These shifts depend upon the mental state and gambling acuteness of the market. Past prices, and the extent to which current requirements have been met at those prices, will influence current purchases. The movement of prices in the present and especially the opinion the market holds as to the probable extent and duration of this price movement will also affect the quantity-taken. Rising prices may stimulate purchases if the rising movement is expected to continue, discourage them if the movement is not expected to be maintained.

This brief analysis leads to the conclusion that sales will be in part the result of price fluctuations and the resulting dealer strategy as well as of the current price. Short-run price variability may not affect total consumption over a fairly long-time period, but the fluctuations in sales and consequent changes in production rates, which are highly important in our contemporary economic structure, must, to some extent, be influenced by this factor.

The work of Evans and Roos, however, has accomplished more than adding emphasis and clarification to the analysis of the effects of price movement offered by previous economists. Even more important is the fact that they adapted the demand equation to the factors of past prices and the rate of change of prices,

⁴ G. C. Evans, "The Dynamics of Monopoly," *American Mathematical Monthly*, XXXI, No. 2 (February, 1934), 77.

and thus laid the foundations for a statistical approach to the study of this type of demand situation. It is not enough merely to state, as did Taussig, that there is a penumbra in which the "demand" relation between quantity and price is inoperative; it is necessary to introduce directly into the demand equation the elements distorting the relation. The equations which Evans and Roos developed for this purpose, and which have been used in the present study, are stated in the following section.

Once the necessity of increasing the complexity of our demand theory to correspond with the complexity of the problem to be dealt with is recognized and steps have been taken toward adjusting the demand theory in this direction, it is possible to state a hypothesis for the demand relation in the short-run steel market. This hypothesis can be worded as follows: "The quantity of steel taken (sales) is a function not only of the present price, but also of the direction and rapidity of the price movement, and of the prices in the past."

Not only is it necessary before beginning a statistical analysis of the steel market to adjust the demand theory, but it is also necessary to consider whether previous statistical techniques, which have been successfully used in the study of the demand for an agricultural commodity, will be suitable for an analysis of steel prices. The procedure for work with agricultural commodities has already been fairly standardized. The student first analyzes, by a study of the economics of the market in question, those factors having some effect on the quantity of the commodity purchased. He then determines which of these have quantitative importance and, after finding a variable which satisfactorily measures the effect of this influence on demand, places this variable in his demand equation. The form of this equation must be carefully considered in the light of his analysis of the market. The equation can then be "fitted" to the data and, from the fit of this equation, a measure can be obtained for the "net" relation between quantity-taken and price. The reliability of the results varies with the accuracy of the data, the validity of the preliminary analysis, and the care taken in its application. A measure of this reliability is given by the size of the statistical measures

of error which are found as a result of fitting the equation to the data.

Moore stated this procedure quite clearly⁵ when he insisted on a statistically measured *ceteris paribus*, but in his study of pig iron⁶ he failed to apply consistently his own theory, for by using no other factors except a simple function of time in deriving his equation of the price-quantity relation, he neglected those factors of change in the business and industrial situation which evidently affect the sales of steel irrespective of the price at which pig iron or steel is offered. This is very probably the basis, as has been pointed out,⁷ for the obtaining of a positive relationship between price and quantity-taken by the use of his technique.

The best solution to the problem is to introduce explicitly those elements in the business cycle which influence the sales of steel. In this way only can the net relation of price and quantity-taken, obscured by these forces much as the net relation of price and quantity-taken of an agricultural commodity was found to be obscured by secular changes, become amenable to statistical measurement and clearer statement.

Consideration of the necessity for taking account of changes in the general business situation as a part of the necessary things to be held "constant" in an analysis of the steel market gives the basis for a second hypothesis of the short run demand for steel. This hypothesis can be stated as follows: "The quantity of steel taken is a function not only of price and of the rate of change of price (as in the first hypothesis), but also of changes in the industrial situation."⁸

Once hypotheses have been formed regarding the relationship of price and sales in the steel market, which are logical and which

⁵ Henry L. Moore, *Economic Cycles: Their Law and Cause* (New York: Macmillan Co., 1914), p. 66.

———, *Forecasting the Yield and Price of Cotton* (New York: Macmillan Co., 1917), pp. 151-62.

⁶ *Op. cit.*, *Economic Cycles*, p. 114.

⁷ P. G. Wright, "Review of Moore's *Economic Cycles*," *Quarterly Journal of Economics* (May, 1915), p. 638.

⁸ In the actual procedure adopted for statistical measurement, as described in the following section, an index of industrial production is used as a first approximation to the joint effect of the various forces of this type operating in the steel market.

afford a reasonably complete description of the forces at work, the next procedure is to test their validity and usefulness by using them to describe the data. In the present study the tests of the two hypotheses were made by putting them in the form of equations and then fitting these equations to monthly data for the sales and price of steel in the three periods—1902-15, 1916-20, and 1921-31. The actual statistical results of these fits are summarized in the concluding section.

Before attempting to summarize these results for those not interested in studying them in detail,⁹ it may be well to state what reliability can be ascribed to them from the statistical point of view. Not only does testing by the statistical measures of error indicate that the direction of the forces affecting the quantity of steel purchases is reliably determined by the study, but the consistency of the results for the two hypotheses further confirms this. Equations expressing both hypotheses, in fact, indicate approximately the same influence of price on sales.¹⁰ Furthermore, results for the different periods show considerable consistency, being qualitatively the same for all the periods and very close quantitatively for the first and last periods. It can be said in general that statistical measurements of reliability indicate the possibility of deriving positive conclusions from the study.

While the two hypotheses indicate approximately the same quantitative effect of price on sales they do not in all respects give an equally satisfactory result. The second hypothesis including the variable measuring the cyclical aspect of business conditions describes the data far more completely than does the first hypothesis.¹¹ This is proof of the fairly evident fact that sales of steel, even in the short run, are affected by factors other than the price of steel alone. Changes in the business situation cannot be disregarded in a study of this character.

The general conclusions for the effect of the variables on quantity of steel taken can be stated as follows:

1. Purchases of steel, as may have been expected, increase and decrease as the general business situation improves or worsens.
2. The movement of price has a very definite and important

⁹ See pages 589 and 591.

¹⁰ See page 593.

¹¹ See page 592.

influence on sales. The result indicates conclusively that rising prices are accompanied by large sales and falling prices by small sales. Such a result was anticipated in the analysis of Evans and in the discussion of the problem already presented.

3. Finally, the study shows that high prices are accompanied by small sales and low prices by large sales. This result is obtained by taking into consideration the factors of the business situation and the direction of the price movement. Where, therefore, it was at one time thought, because of observed concomitances, that the demand relationship was "positive,"¹² the true net relationship between the two variables is of the typical negatively sloping demand curve sort. While statistical results do not afford positive proof that this is a demand relationship precisely of the classical sort, or that results obtained are not influenced by factors which the statistical techniques have neither eliminated nor taken into account, the quantitative results are at least consistent with a reasonable demand theory.

These results are, unfortunately, inadequate to give an accurate measure of the elasticity of demand, which varies a great deal between boom and depression. They do show, however, that further analysis should yield measures of this elasticity.

The contributions which are claimed for the study are the following: First, it demonstrates not only the usefulness, but the indispensability of taking account of a complex price behavior in analyzing this type of market. The equations of Evans and Roos thus have a practical as well as a theoretical value. Second, the study demonstrates the possibility of a successful extension of the statistical techniques employed by the agricultural economists into the field of producers' goods. Third, it measures for the first time to a first approximation a relationship which can be described as the "demand for steel."

The limitations of the study are of two types: first, limitation of method, and, second, limitation of application. The limitation of the method is principally due to the imperfection of the statistical technique employed, namely that of multiple correlation.¹³

¹² Henry L. Moore, *Economic Cycles*, *loc. cit.*, p. 114.

¹³ See pages 593-94.

The second type of limitation covers a restriction in the number and complexity of hypotheses used. The two hypotheses tested in the study are not the only ones which might be suggested, and experiments with variations in, and additions to, the hypotheses might give results which would be quantitatively more satisfactory. This study is only a beginning in a new field, but it demonstrates that progress is possible.

II

The actual equations used and quantitative results obtained in the analysis will now be presented. No attempt will be made to give a detailed discussion of the statistical problems involved, nor will the economics of the problem be advanced beyond the discussion already given.

The equations for the first hypothesis are as follows:

$$A \quad y = ap + b + h \frac{dp}{dt} \quad (1)$$

$$(1) \quad y(t) = ap(t) + b + \int_{-\infty}^t \varphi(t-\tau) p(\tau) d\tau \quad (2.1)$$

$$B \quad (2) \quad y(t) = ap(t) + b + a_1 p(t-r) + a_2 p(t-2r) + \dots + a_k p(t-kr) \quad (2.2)$$

$$(3) \quad y(t) = ap(t) + b + \sum_{-\infty}^t w_i p(\tau) \quad (2.3)$$

$$C \quad y(t) = ap(t) + b + h \frac{dp}{dt} + \sum_{-\infty}^t w_i p(\tau) \quad (3)$$

Equation (1) where y = quantity demanded, p = price, dp/dt = the rate of change of price with respect to time, was first advanced by Evans in 1924.¹⁴ This provides for the influence on sales of rising and falling prices.

Equations (2.1), (2.2), and (2.3) express the dependence of sales on prices in the past. Equation (2.1) is a Volterra integral equation

¹⁴ G. C. Evans, "The Dynamics of Monopoly," *American Mathematical Monthly*, XXXI, No. 2 (February, 1924), 77.

which was developed by Roos¹⁵ and in which $p(-\infty)$ is assumed finite, and $\varphi(t-\tau)$ is negligible if $t-\tau$ is large and negative. A more simple and less elegant equation (2.2) expresses the same dependence and is easier to fit to the data. It can also be expressed by the summation term, as in equation (2.3) in which w_i is the weight to be given the different prices in the past. This equation can, of course, be combined with equation (1) giving

$$(3) \quad y(t) = ap(t) + b + h \frac{dp}{dt} + \sum_{-\infty}^t w_i p(\tau)$$

which is equation (3).

The second hypothesis is expressed by the equation

$$(4) \quad y = ap + b + h \frac{dp}{dt} + cI + dt$$

where y is the quantity-taken, p is the price, dp/dt is, as before, the rate of change of price with respect to time, I is an Index of Industrial Production, and t is time. An Index of Industrial Production, as already noted, is used as a most convenient summary of the business cycle conditions creating fluctuations in the purchases of steel. The use of time as an explicit variable is for the purpose of excluding the problem of the effect of trends in the price of steel on the purchases of steel.

The price used in fitting the equations of both hypotheses is the composite price for finished steel by the *Iron Age*. The Index for Industrial Production selected for the first two periods is the American Telephone and Telegraph Index of Business, and for the third period the Standard Statistics Index of Industrial Production.

The figure used for the quantity-taken is, in each case, an Index of Sales. This is used as a substitute for production figures, which are unsatisfactory because of the fact that production is on order and that production in a certain month does not necessarily coincide from the standpoint of sales with the price of that month. The construction of this Index of Sales is made by taking account

¹⁵ C. F. Roos, "Dynamical Theory of Economics," *Journal of Political Economy*, XXXV (1929), 632-56.

of the changes in orders of the United States Steel Corporation as well as figures for current production. It represents to a first approximation the quantity of steel actually sold during a given period.¹⁶

The use of this Index of Sales is as necessary to the securing of reasonable results as the improved hypotheses already described. The fitting of equations representing the two hypotheses to production data gives less consistent and satisfactory results than those obtained with the use of the Index of Sales. At the same time the substitution of the Index of Sales for production figures does not make improvement in the demand hypothesis unnecessary. The study indicated that both the extension of the demand hypothesis and the revision of the data were required. The equations customary in agricultural demand studies still give positive quantity-price relationships when the Index of Sales is used.¹⁷

The equations were fitted to the monthly data from 1902 to 1930 by the three periods—1902–15, 1916–20, 1921–30. This division was made in order to separate the periods of extreme price fluctuation from the more “normal” ones.

The testing of the first hypothesis must be done by the fitting of three of the equations representing it. The hypothesis may be taken in the form expressed by equation (1), i.e.,

$$(1) \quad y = ap + b + h \frac{dp}{dt}$$

¹⁶ This index has been criticized on the ground that it includes speculative elements, the pyramiding of orders when prices are rising and cancellation when prices are falling. This type of activity in the steel market, however, has already been taken into account by inclusion of a rate of change of price term in the equations. The hypothesis is thus made to take proper account of an element in the market which cannot be separated from the data.

¹⁷ For example, the fitting of the equation $y = ap + b + ct$ where y = quantity demanded, p = price, and t = time, when the series used for “quantity-demanded” is the Index of Sales, gives the following parameters for p :

—0.11 ± 0.41 in the period 1902–15

—0.35 ± 0.16 in the period 1916–20

0.095 ± 0.42 in the period 1921–30

These parameters, expressing the “net” relation of price and quantity—taken after the elimination of trend, while not consistently positive as in earlier studies using production data, are still so inconsistent and so small in relation to their standard errors that interpretation is not possible.

or in its second form (2.3), i.e.,

$$(2.3) \quad y(t) = ap(t) + b + \sum_{-\infty}^t w_i p(\tau)$$

or in its complete form in (3):

$$(3) \quad y(t) = ap(t) + b + h \frac{dp}{dt} + \sum_{-\infty}^t w_i p(\tau) .$$

Before fitting these equations, the substitution of discrete prices for an integral as an expression for prices in the past, must be paralleled by a substitution of differences for the differential as an expression of the rate of change of price. The effect of price movement can operate in the market only through actual differences in price over a discrete time interval or through a marked trend of prices in the recent past. Analysis of the relation of sales to price movements indicated that the general movement in prices had more influence in determining sales than the month-to-month fluctuations. Consequently, the price series was smoothed by a five months' moving average, and the first differences of this smoothed series were used as a measure of the rate of change of price in fitting the equations.

In selecting the prices in the past for the fitting of the summation term the effect of remote prices was disregarded and the prices of the third, sixth, and ninth months preceding the one in question were used. Remote prices should be disregarded in spite of possible high correlations between the past sales and these prices, since these high correlations are due to the juxtaposition of two series having cycles of approximately the same length. Only three months were selected because the use of price in adjacent months would make the coefficients so unstable that no satisfactory conclusion could be drawn. Equations were fitted to the trend deviates of the data. Table I gives the results of the fitting of the three equations.

It is to be noted that only equation (3) gives a consistent negative relationship between the price and quantity-taken, which is

TABLE I
THE INDEX OF SALES* FOR STEEL AS A FUNCTION OF PRICES,† THE RATE OF CHANGE OF PRICE,‡
AND PREVIOUS PRICES, BY PERIODS, 1902-30

Period and Type of Equation	a	b	h	a_1	a_2	a_3	R'§	ε
$y(t) = ap(t) + b + hp'$								
1902-15.....	-1.00 ± 0.32¶	3.64	3.19 ± 0.28				0.66	0.47
1916-20.....	-0.38 ± 0.12	4.61	0.63 ± 0.11				0.65	0.62
1921-30.....	0.60 ± 0.29	0.90	3.15 ± 0.26				0.74	0.57
$y(t) = ap(t) + b + a_1p(t-3)$ $+ a_2p(t-6) + a_3p(t-9)$								
1902-15.....	2.66 ± 0.56	5.81		-3.80 ± 0.91	0.41 ± 0.95	-1.41 ± 0.63	0.58	0.51
1916-20.....	0.09 ± 0.10	5.45		-0.48 ± 0.23	-0.24 ± 0.17		0.56	0.68
1921-30.....	2.97 ± 0.68	2.94		-2.22 ± 0.80	-0.49 ± 0.67	-0.46 ± 0.43	0.68	0.62
$y(t) = ap(t) + b + hp' + a_1p(t-3)$ $+ a_2p(t-6) + a_3p(t-9)$								
1902-15.....	-2.50 ± 1.00	4.58	3.78 ± 0.64	2.06 ± 1.28	0.02 ± 0.86	-1.07 ± 0.57	0.68	0.45
1916-20.....	-0.60 ± 0.25	4.56	0.78 ± 0.21	0.49 ± 0.33	-0.26 ± 0.15		0.67	0.62
1921-30.....	-1.93 ± 1.09	-0.68	5.39 ± 0.99	3.57 ± 1.28	-1.26 ± 0.61	0.83 ± 0.40	0.75	0.56

* y = sales in millions of tons. (Data for first and last periods corrected for trend.)

† p = price in cents per pound. (Data for first and last periods corrected for trend.)

‡ p' is the rate of change of price. (First differences of smoothed price quotations.)

§ R' is the coefficient of multiple correlation corrected for the number of parameters.

|| ε is the standard error of estimate corrected for the number of parameters.

¶ Standard error of the parameter.

the only theoretically reasonable relationship between these two quantities.¹⁸ It is quite clear, however, that a determination of demand at a moment is largely due to the rate of change of price immediately previous; there can be little doubt that the movement of price definitely influences sales. Further inspection of Table I demonstrates that the change in sign between the price of the past month and sales in equations (2.3) and (3) is due to the inclusion of the differential term in the latter equation. The reason for this is that giving a positive fit to the price in the current month, and a negative fit to that of three months previous, which is the result of fitting equation (2.3), means that rising prices are weighted positively and falling negatively. If the prices are about the same the weights will cancel out. A positive coefficient for the first difference of a price series, however, amounts to the same thing, for in this case also rising prices are weighted positively, and falling prices negatively. In other words, in equation (2.3) the weighting given the two prices takes the place of the differential term.

From the fit of equation (3), however, the following definite conclusion can be drawn. When the bullish and bearish effects of rising and falling prices are held constant, a price lower than the price three months previous stimulates sales, while a higher price dampens them. In other words, when prices have fallen, sales will be stimulated, and when they have risen, sales will be depressed. It is, therefore, possible to conclude that when constant weights are given to the rate of change of price and previous prices, the lower the current price the greater will be the quantity taken and vice versa.

This hypothesis, however, is not completely satisfactory. Not only is it incomplete, as we have already noted, but the statistical

¹⁸ It is not theoretically impossible to have a positively sloping demand curve. This can occur when goods are competing rather than completing and the expenditure of the article consumed is a large portion of the budget. The familiar example of such a situation is that of potatoes in Ireland, but this type of situation does not apply to steel or any other important producers' good. Therefore, it is not unreasonable to assume upon the basis of the usual analysis that the demand curve must be negatively inclined.

results might be better. In fact, the size of the coefficients of multiple correlation (0.63-0.75) indicates that there are important elements in the quantity-price relationship for which no explanation has been given. Besides this, it is probable that the differential term is given an importance which is more properly attributed to another variable, namely, the changes in the general business situation.

An additional point in favor of the second hypothesis is the greater simplicity in the fitting of the equation (4)

$$(4) \quad y = ap + b + h \frac{dp}{dt} + cI + dt$$

where y is the quantity-taken, p is the price, dp/dt is as before the rate of change of price with respect to time, I is the Index of In-

TABLE II

ANALYSIS OF THE DEMAND FOR STEEL, 1902-30, BY EQUATION:*

$$y = b + ap + hp' + cI + dt$$

 (The figures preceded by $\pm$ are standard errors)

Period	$y = f(p, p', I, t)$	$R'^{\dagger}$	$\epsilon^{\ddagger}$	$yp'p'I t^{\S}$
1902-15.	$y = 4.20 - (1.56 \pm 0.31)p + (7.99 \pm 1.06)p' + (0.036 \pm 0.005)I + (0.41 \pm 0.12)t$	0.81	0.45	-0.51
1916-20.	$y = 3.84 - (0.55 \pm 0.08)p + (0.48 \pm 0.21)p' + (0.12 \pm 0.011)I + (0.15 \pm 0.04)t$	0.88	0.39	-0.68
1921-30.	$y = 1.49 - (1.27 \pm 0.23)p + (6.27 \pm 0.94)p' + (4.64 \pm 0.35)I + (0.03 \pm 0.17)t$	0.92	0.41	-0.53

* y = Index of sales of steel, millions of gross tons.

p = Price of steel, cents per pound.

p' = First differences of price. (Written as dp/dt in the text.)

I = Index of Industrial Production, 1902-20: American Telephone & Telegraph Index of Business, plus and minus deviations.

1921-30: Standard Statistics Index of Industrial Production, ratios to normal.

t = Time (0.01 is monthly interval), origin at first month in period.

$\dagger R'$ is the coefficient of multiple correlation corrected for the number of parameters.

$\ddagger \epsilon$ is the standard error of estimate corrected for the number of parameters.

$\S y p' p' I t$ is the coefficient of part correlations. See M. J. B. Ezekiel, *Methods of Correlation Analysis*. (New York: John Wiley & Sons, Inc., 1930), pp. 181-83. Defined generally, it is $r_{x_1 x_2 \cdot x_3 \cdot x_4}$

where $x_{2 \cdot 345} = x_2 - e_{2 \cdot 345}$

dustrial Production and t is time. The results of the fitting appear in Table II.

An analysis of the tabulation suggests the following conclusions:

1. The estimates of the Index of Sales from the equations are fairly good, and the size of the coefficients of multiple correlation precludes the possibility that the relationships are due to chance.

2. The relations described by the coefficients of regression are uniform and significant, and indicate no change in direction from period to period. The net relation between quantity-taken and price is negative, between quantity-taken and the rate of change of price positive, that between quantity-taken and an Index of Industrial Production positive, and that between demand and time generally positive. There is some change in the order of the coefficients for the war period, but the use of the logarithms of price would probably have reduced the difference. The coefficients of the indices of production are, unfortunately, not comparable. There is some indication, however, that the relations between price and the rate of change of price returned in the third period to their pre-war status.

3. Negative relationship between quantity-taken and price can be measured by a part correlation coefficient, or a partial correlation coefficient, as well as by the regression coefficient. The part correlation coefficient¹⁹ is indicated in Table II. It measures the relationship between price and quantity-taken, after the latter has been corrected for the rate of change of price, industrial production, and time, by giving these three variables their weights in the regular regression equation.

The third hypothesis, as might be expected, gives a better fit than the second. For each period and each equation the coefficients of correlation corrected for the number of constants are:

Period	First Hypothesis Equation (3)	Second Hypothesis Equation (4)
1902-15.....	0.68	0.81
1916-20.....	0.67	0.88
1921-30.....	0.75	0.92

¹⁹ M. J. B. Ezekiel, *Methods of Correlation Analysis* (New York: John Wiley & Sons, Inc., 1930), pp. 181-83.

The results of fitting the two equations give interesting relationships. In each equation, the relation between sales and the rate of change of price is positive and significant. Either demand pattern substantiates the bullish and bearish effect of rising and falling prices. But the outstanding feature is the stability of the coefficients of the price term in the two cases. A comparison of the coefficients of price for the three periods indicates the following:

Period	First Hypothesis Equation (3)	Second Hypothesis Equation (4)
1902-15.....	-2.50 ± 1.00	-1.56 ± 0.31
1916-20.....	-0.60 ± 0.25	-0.55 ± 0.08
1921-30.....	-1.93 ± 1.09	-1.27 ± 0.23

In none of the three cases is the difference between the coefficients greater than the larger of the standard errors. This stability, of course, does not prove that quantity and price would have a relation of this order under all possible demand hypotheses, but it does indicate that relations obtained by fitting the two equations are more than a chance phenomenon.

III

There now exist by the criteria of rationality, goodness of fit, reasonableness of sign and size of the parameters, two fairly good demand equations for steel. It is possible to conclude that if either of two different sets of factors is given constant weights, the relation between price and quantity is negative and for both sets approximately of the same order.

The hypotheses made, however, are by no means the only possible ones. As already suggested, the testing of more complex and flexible hypotheses might change the results obtained. The general certainty of the conclusions must also be qualified by the fact that the application of a multiple correlation technique gives only an average result for the period covered. It is possible for the general configuration in time of the series, all of which have cycles of approximately the same length, to produce the results obtained. For example, the Index of Industrial Production typically moves between the Index of Sales and the price of steel. The inclusion of this series in the equation, by its very position in rela-

tion to the other two series, tends to make their net relationship, as measured by the equation, a negative one.

This means that the general significance of the coefficients is limited. There is only one observation for each month, only one set of observations for each cyclical movement. The observations for each cyclical movement, furthermore, follow much the same pattern. This limited pattern of observations limits the generality of the coefficients we derive by fitting equations to these observations. Thus there is no proof that the relations given by the coefficients may not be irreversible. That is, it is not certain, for example, whether a rise of prices early in the boom would have the same effect as one late in the boom. Such a contingency did not arise within the range of the observations. The fitted equations are, then, a description of the average, though fairly uniform, relationships between the series. It is quite possible that the action of other unanalyzed factors produces the observed concomitances. An attempt to analyze the possible effect of the position of the cyclical movement on the reaction of the quantity-taken to price and to movement of price will be a pertinent subject for research.

Despite these qualifications, this study, although a beginning, seems to be successful in part in deriving a plausible analysis from the concrete facts of price behavior for this type of commodity. It is reasonable to believe it may be a departure for a more rigorous and comprehensive analysis of price and quantity relationships.

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THE STATISTICAL LAW OF DEMAND FOR A PRODUCER'S
GOOD AS ILLUSTRATED BY THE DEMAND FOR STEEL

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THE STATISTICAL LAW OF DEMAND FOR A PRODUCER'S GOOD AS ILLUSTRATED BY THE DEMAND FOR STEEL

By ROSWELL H. WHITMAN

ALTHOUGH H. L. Moore attempted a statistical analysis of the demand for producers' goods¹ very shortly after he had studied the demand for agricultural commodities with some success, up to the present no satisfactory work in the former field has been done. The modicum of success which has attended the work of the agricultural economists has not been duplicated in the field of producers' goods because of the greater complexity of the latter field and the lack of adequate theoretical and statistical technique for dealing with it.

The present article presents the procedure and results of a statistical study of the demand for steel, which is an important member of the group of "producers' goods." In the first section, I shall state certain hypotheses for the demand function for steel in the form of equations, and then briefly explain their theoretical implications. In the following sections, I shall describe the fitting of the equations to the data and discuss the results obtained.

1. THE THEORETICAL FORMULATION

There are different hypotheses which might be used to explain the demand behavior of steel. The following equations state these hypotheses in a mathematical form:

$$(1) \quad y = ap + b + ct.$$

$$(2) \quad y = ap + b + h \frac{dp}{dt}.$$

$$(3) \quad y(t) = ap(t) + b + \int_{-\infty}^t \phi(t - \tau)p(\tau)d\tau.$$

$$(4) \quad y(t) = ap(t) + b + a_1p(t - r) + a_2p(t - 2r) \\ + \dots + a_kp(t - kr) = ap(t) + b + \sum_k a_kp(t - kr).$$

$$(5) \quad y(t) = ap(t) + b + h \frac{dp}{dt} + \sum_k a_kp(t - kr).$$

$$(6) \quad y = ap + b + h \frac{dp}{dt} + cI + dt.$$

The hypothesis expressed by equation (1), where y =quantity demanded, p =price, and t =time, makes the quantity demanded a simple

¹ *Economic Cycles: Their Law and Cause*, New York, 1914.

function of the price of the commodity and time. It implies that the "demand" relation of quantity and price is disturbed only by the trends of the two series. This equation is frequently used in the study of the demand for an agricultural commodity.

The hypotheses, equations (2)–(5), are the result of the work of Evans and Roos. Equation (2), where dp/dt , or p' , is the rate of change of price with respect to time, was first advanced by G. C. Evans in 1924.² In the equation the constants should have the signs $a < 0$, $b > 0$, h not determined but probably > 0 . The term involving p' was introduced "in order to take account of the fact that the demand for a commodity may depend on the rate of change of price (whether, for instance, it is increasing or decreasing) as well as the price itself."³ Evans believed that in the practical case the sign of the coefficient of the derivative was positive.

This reaction of sales to rising and falling prices had been noted by the classical economists,⁴ but they had never attempted to take account of this disturbing factor directly in the demand equation. There is little doubt that the steel market is an example of this type of demand behavior. The supply of steel is never fixed and there can be immediate and pronounced shifts in the quantity-taken. These shifts depend upon the mental state and gambling acuteness of the market. Past prices, and the extent to which current requirements have been made at those prices, are also important. The amount purchased in the present may depend not only on the price at that time, but upon the movement of price at that time.

The second element in the steel market is the movement of prices in the present and the opinion the market holds as to the probable extent and direction of this movement. Rising prices may stimulate purchases, if the rising movement is expected to continue, discourage them if the movement is not expected to be maintained. The briefest analysis leads to the conclusion that consumption will be the result of the strategy of dealers in a shifting market, as well as the result of the present price. Thus, it is fairly certain that the application of equation (2) will be pertinent to the problem.

Not only may the rate of change of price effect demand, but, as already noted, past as well as present prices may influence it. Demand

² "The Dynamics of Monopoly," *American Mathematical Monthly*, Vol. 31, No. 2, February 1924, p. 77.

³ G. C. Evans, *Mathematical Introduction to Economics*, New York, 1930, p. 143.

⁴ Alfred Marshall, *Principles of Economics*, 8th Edition, London, 1920, Book III, Chapter IV, p. 112.

F. W. Taussig, "Is the Market Price Determinate?" *Quarterly Journal of Economics*, Vol. 35, 1921, pp. 394–411.

may, in fact, depend upon prices which prevailed indefinitely into the past. Upon this assumption, says C. F. Roos,⁵ the equation of demand becomes a Volterra integral equation in the form of equation (3), $p(-\infty)$ is assumed finite, and $\phi(t-\tau)$ is negligible if $t-\tau$ is large and negative. This form of the equation is very satisfactory to the mathematician and can be used for theoretical analysis, but for the purpose of the statistician the approach of Evans is the more convenient. He writes the equation in the form of equation (4), where $a_1, a_2, \dots, a_k$, form a sequence of constants decreasing in numerical value, and r is a suitable interval of time, say a week or a month; in this way the demand would depend on the price one week, two weeks, etc., before, as well as on the price at that time. We might even make $y(t)$ depend on the price at all times. This equation can, of course, be combined with the one using the rate of change of price term as is done in equation (5).

Roos has suggested a form that is even more general. He says: "It seems incorrect, except in special cases, to write the demand for a commodity as a function of the prices alone of the commodity, or even as a function of the price of several commodities. The demand depends upon the price, the rate of change of price, and upon the accumulation of these effects. In some cases it may even depend upon the rate of production, the acceleration of the rates of production, and upon the cumulation of these effects. The general demand equation must then have this form,

$$G(u_1, u_1', \dots, u_n, u_n', p, p', t) = \int_0^t p(u_1, u_1', \dots, u_n, u_n', p, p', t) dt."$$
⁶

This attempt to express demand in as general a form as possible is desirable. In actual investigations, however, the statistician must be satisfied with approximations. Some advance in the generality of demand studies will be obtained if satisfactory use can be made of the limited equation (5).

The last hypothesis is expressed by equation (6) where I is an index of industrial production, and t is time. This hypothesis is somewhat broader than the first and takes account of forces influencing the behavior of the price and production of steel, which an analysis such as that of Moore's had neglected. It is very likely, in fact, that the consumption of steel would vary even though the price did not change. The forces causing this independent variation are usually important elements in the fluctuations known as the business cycle. The demand equation, therefore, should be extended to include these factors, whose fluctuations would otherwise obscure the true relation of quantity and price.

⁵ "Dynamical Theory of Economics," *Journal of Political Economy*, Vol. 35, 1929, pp. 632-656.

⁶ Roos, *op. cit.*, p. 654.

An index of industrial production is the most convenient summary of the business cycle forces creating fluctuations in the purchases of steel. While other factors may be more important in a causal sense, an equation embodying all the possibilities would be exceedingly complex. It is, therefore, convenient to assume that an index of industrial production is a resultant of these forces and that its movement describes the shift of demand through time.

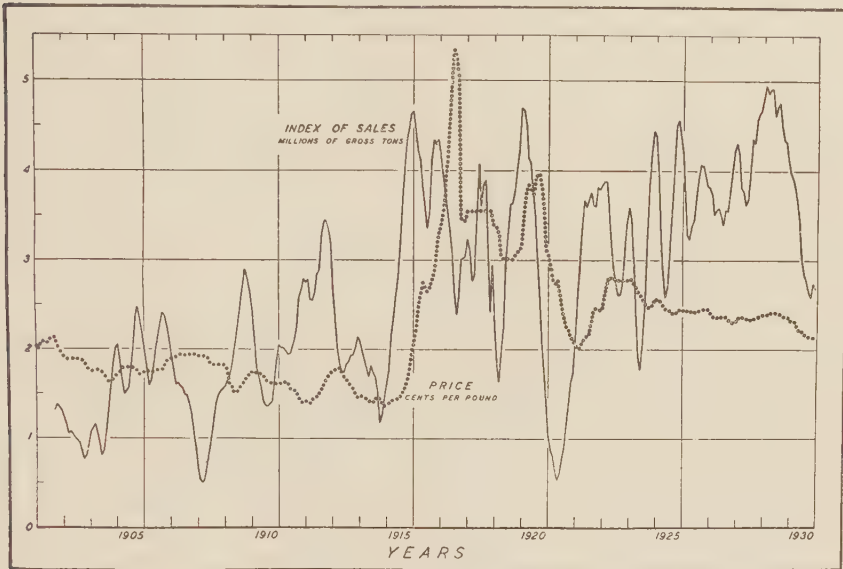


FIGURE 1.—Index of Sales of Steel, and Composite Price of Finished Steel, 1902–1930⁷

It should be noted that time is included as an explicit variable; it acts as a catch-all for the linear trend factors only. Its use is not a denial that the trends in the price of steel may influence the purchases of steel, but an attempt to exclude this problem from that of the cyclical behavior of purchases, which is the field of the present investigation.

The data used for the testing of the hypotheses are an Index of Sales of steel for the period 1902 to 1930, and composite prices for finished steel by the *Iron Age* for the same period. The Index of Sales is a substitute for production figures which are unsatisfactory. The reason for its choice and details of its construction are given in Appendix A. Figure 1 shows the two series for the year 1902–1930.

⁷ Source of data: Index of Sales, Appendix A; Price, *Iron Age*, Vol. 127, 1931, p. 133.

For the purposes of fitting the equations, the data were divided into three periods 1902–1915, 1916–1920, 1921–1930. This division was made in order to separate the period of extreme price fluctuation from the more “normal” ones. There is some question whether the use of such periods is in line with the interpretation given their equations by Evans and Roos. Roos thinks that these equations should be applied to a fairly short period in which the movement of prices is in one direction. At the end of an upswing or a downswing, the general solution of equations describing the total economic situation will change, and presumably, the demand equation will be altered.⁸ Evans was of the same opinion,⁹ but there is in Evans’ later statement¹⁰ some justification for the fit to the longer period. He suggested that if crises were not violent, it was possible to have a period of more than one cyclical swing in prices expressed by the same equation. The facts of the case do no great violence to these conditions, and the consistency of the inter-relations suggests the selection of periods made.

2. FITTING OF THE SIMPLE DEMAND EQUATION

The first hypothesis expressed by equation (1), in which the constants will have the signs $a < 0$, $b > 0$, c not determined, did not prove to be an adequate one. The results are as follows: In the period 1902–1915,

$$y = 1.35 - (0.11 \pm 0.41)p + (0.90 \pm 0.15)t;$$

in the period 1916–1920,

$$y = 4.93 - (0.35 \pm 0.16)p - (0.12 \pm 0.06)t;$$

and in the period 1921–1930,

$$y = 2.05 + (0.095 \pm 0.42)p + (1.80 \pm 0.24)t;$$

the sign of p is twice negative and once positive and in only one period is it significant. Such results are too shifting to be interpreted, and, as might be expected from the logic of the hypothesis, it cannot give a satisfactory picture of the demand for steel.

3. FITTING OF THE EVANS-ROOS EQUATIONS.

The Evans-Roos theory of demand is of more use. The testing of it divides itself into three steps. There is the hypothesis in its first form expressed by equation (2), in its second form by equation (4), and in its complete form by equation (5).

⁸ C. F. Roos, “A Mathematical Theory of Price Fluctuations,” *Journal of Political Economy*, Vol. 38, October 1930, p. 513.

⁹ G. C. Evans, Correspondence with Prof. Schultz.

¹⁰ “A Simple Theory of Economic Crises,” *Journal of the American Statistical Association*, Vol. 26, March Supplement, p. 67.

The fitting was done as follows: An approximation to the differential term was made by taking the first differences of a price series smoothed by a five-month moving average. In fitting the summation term the effects of remote prices can be disregarded, and the prices of the third, sixth, and ninth months preceding the one in question used.

Inspection of the graph indicates that sales are negatively correlated with past prices. This negative correlation extends to prices considerably remote (more than a year), and in some cases (the third period) sales are apparently more closely related to remote prices than to those of current months. There is reason to believe, however, that this relation is more apparent than real. The graph indicates that the Index of Sales leads prices by approximately ten months, or a quarter period, the whole period being the length of the cycle, about forty months. This means that ten months later prices will be in the same phase of cyclical movement as sales are in any current month and in exactly opposite phase ten months previous, with the result that sales must be negatively correlated with prices ten months previous. In other words, the observed correlations may be the result of the particular timing of the two series in question. As a result of this analysis, it seems unsafe to include prices further in the past than nine months.

Moreover, since the use of prices in adjacent months would make the coefficients so unstable that no satisfactory conclusions could be drawn, only three prices of the nine possible ones were selected. In the second period, the price of the nine months previous was not used because the period was shorter and the correlation between sales and past prices fell off rapidly. The equations were fitted to the trend deviates of the data.

There is one more problem concerning the fitting of these equations. Equation (5) includes both the differential term and the summation term. In a sense, however, both terms measure the same phenomena, and if the price in the month immediately preceding was used, there would be no reason for the use of the rate of change term. However, since the first price in the past used was that of three months previous, it can be argued that using the two terms causes no serious difficulties. It will be seen, subsequently, that equation (5) is the only form of the Evans-Roos equations giving a reasonable fit. Table I gives the results of fitting the three equations. The outstanding points to note are listed below.

In equation (2):

1. The differential or rate of change term, as measured by the size of the b term, is the most important in the estimation of sales; and
2. The sign of the coefficients of price is negative in two cases and positive in one.

TABLE I.—THE INDEX OF SALES^a FOR STEEL AS A FUNCTION OF PRICES,^b THE RATE OF CHANGE OF PRICE,^c AND PREVIOUS PRICES, BY PERIODS, 1902-1930

Type of Equation and Period	a	b	h	a_1	a_2	a_3	R^2 ^d	ϵ ^e
$y(t) = ap(t) + b + hp'$ Eq. (2) 1902-1915 1916-1920 1921-1930	-1.00 ± 0.32 ^f -0.38 ± 0.12 0.60 ± 0.29	3.64 4.61 0.90	3.19 ± 0.28 0.63 ± 0.11 3.15 ± 0.26				0.66 0.65 0.74	0.47 0.62 0.57
$y(t) = ap(t) + b + a_1p(t-3)$ + $a_2p(t-6) + a_3p(t-9)$ Eq. (4) 1902-1915 1916-1920 1921-1930	2.66 ± 0.56 0.09 ± 0.19 2.97 ± 0.68	5.81 5.45 2.94		-3.80 ± 0.91 -0.48 ± 0.23 -2.22 ± 0.80	0.41 ± 0.95 -0.24 ± 0.17 -0.49 ± 0.67	-1.41 ± 0.63 -0.46 ± 0.43	0.58 0.56 0.68	0.51 0.68 0.62
$y(t) = ap(t) + b + hp' + a_1p(t-3)$ + $a_2p(t-6) + a_3p(t-9)$ Eq. (5) 1902-1915 1916-1920 1921-1930	-2.50 ± 1.00 -0.60 ± 0.25 -1.93 ± 1.09	4.58 4.56 -0.68	3.78 ± 0.64 0.78 ± 0.21 5.39 ± 0.99	2.06 ± 1.28 0.49 ± 0.33 3.57 ± 1.28	0.02 ± 0.86 -0.26 ± 0.15 -1.26 ± 0.61	-1.07 ± 0.57 0.83 ± 0.46	0.68 0.67 0.75	0.45 0.62 0.56

^a y = sales in millions of tons. (Data for first and last periods corrected for trend.)

^b p = price in cents per pound. (Data for first and last periods corrected for trend.)

^c p' = rate of change of price. (First differences of smoothed price quotations.)

^d R^2 is the coefficient of multiple correlation corrected for the number of parameters.

^e ϵ is the standard error of estimate corrected for the number of parameters.

^f Standard error of the parameter.

In equation (4):

1. Though some of the coefficients are fairly stable, the fit is not as good as in the first case;
2. The signs of a_1 , a_2 , a_3 are negative and generally decreasing; and
3. The sign of a is positive.

In equation (5):

1. The sign of the coefficient of p' is always positive, and this variable is usually the most important in the estimation of sales;
2. The coefficients of the terms representing past prices are of different signs, but that of $p(t-3)$, i.e., a_1 , is always positive; and
3. The coefficient of $p(t)$, or a , is always negative and significant.

The size of the standard errors is due to the large correlations between the various prices. Many of the terms are not significant but will be tentatively retained to aid in the interpretation.

The results of the Evans-Roos hypotheses are difficult to interpret due to the variation in the coefficients obtained by using different forms of the equations. The interpretation is further limited by the fact that we do not have a sufficiently precise a priori hypothesis for the *direction* in which past prices act. It is clear that past prices should have some effect on present sales, but we are not told what the signs of a_1 , a_2 , a_3 , should be, or *what changes in them the introduction of p' should make*. It is certain, furthermore, that the coefficient of p should be negative ($a < 0$), and it is probable that the coefficient of p' should be positive ($h > 0$). It is well to know this, but knowledge of it is not sufficient to prevent a considerable number of explanations from fitting the facts equally well. I shall state one and make it my tentative conclusion regarding the Evans-Roos theory.

The most simple and important generalization to be drawn from the results of the fitting is that the determination of demand *is due largely to the differential term*. There can be no doubt that the movement of prices definitely influences sales. Comprehension of this fact helps in the interpretation of the remaining coefficients.

The requirement of $a < 0$ shows equation (2), with the differential term only, to be inadequate. As a result of fitting this equation the sign of a was twice negative and once positive. It seems difficult to find meaning in either a positive a or its instability of sign.

Careful inspection of the relative importance of the coefficients obtained in fitting equations (4) and (5) will assist the interpretation of these variations.

The difference in the relation of the coefficients of $p(t)$ and $p(t-3)$, i.e., a and a_1 , in the two cases is striking. In the first instance that of $p(t)$ is positive and that of $p(t-3)$ negative, while in the second the opposite is true. The change must be due to the introduction of p'

in the second case, but why should such a difference arise? If a positive weight is given the price in the current month, and a negative weight to that of the three months previous, it signifies that rising prices are weighted positively and falling prices negatively. If the prices are about the same the weights will cancel out. A positive coefficient of a differential term, however, amounts to the same thing, for in the latter case also rising prices are weighted positively and falling prices negatively. In other words, in the first case, the weighting of the two prices took the place of the differential term. In the second case the weighting of the present price and of that three months previous may have economic meaning.

It appears from the fitted equation that when the bullish and bearish effects of rising and falling prices are held constant, a price lower than the price three months previous stimulates sales, while a higher price dampens them. This would be partly the result of purchases withheld during the period of falling prices entering the market. There is a possible interpretation, then, that the market rises and falls with rising and falling prices, but that when prices *have* fallen sales will be stimulated, and when they *have* risen sales will be depressed. It is, therefore, possible to conclude that if constant weights are given the rate of change of price and previous prices, the lower the current price the greater will be the quantity-taken, and vice versa.

This hypothesis, however, is not completely satisfactory. Not only is it incomplete, as we have already noted, but the statistical results are not good, taken in their totality.

4. THE FITTING OF THE GENERAL BUSINESS EQUATION

It is probable that the differential term in the Evans-Roos equations is given an importance which is more properly attributed to another variable, namely, *the changes in the general business situation*. It may therefore be better to try an equation in the form of equation (6), where I is the index of industrial production and t is time. For the index of industrial production, the American Telephone and Telegraph Index of Business is used in the first two periods, the Standard Statistics Index of Industrial Production in the last. The equation is fitted to the actual data. In the A. T. & T. index the trend has been removed, while the Standard Statistics index includes the trend. The results of the fitting appear in Table II.

In summarizing the results, the most important relationships established are:

1. The estimates of the Index of Sales from this equation are fairly good.
2. The relations described by the coefficients of regression are uni-

TABLE II.—ANALYSIS OF THE DEMAND FOR STEEL, 1902-1930, BY THE GENERAL BUSINESS EQUATION^a
(The figures preceded by $\pm$ are standard errors.)

Period	$y = b + ap + hdp/dt + cl + dt$	R'^b	ϵ^c	$y p'' dp/dt I t^d$
1902-1915	$y = 4.20 - (1.56 \pm 0.31) p + (7.99 \pm 1.06) dp/dt$ $+ (0.036 \pm 0.005) I + (0.41 \pm 0.12) t$	0.81	0.45	-0.51
1916-1920	$y = 3.84 - (0.55 \pm 0.08) p + (0.48 \pm 0.21) dp/dt$ $+ (0.12 \pm 0.011) I + (0.15 \pm 0.04) t$	0.88	0.39	-0.68
1921-1930	$y = 1.49 - (1.27 \pm 0.23) p + (6.27 \pm 0.94) dp/dt$ $+ (4.64 \pm 0.35) I - (0.03 \pm 0.17) t$	0.92	0.41	-0.53

^a y = Index of Sales of Steel, millions of gross tons.

p = Price of Steel cents per pound.

dp/dt = First differences of Price.

I = Index of Industrial Production, 1902-1920: A. T. & T. Index, plus and minus deviations, 1921-1930: Standard Statistics Index of Industrial Production—ratios to normal.

t = Time (0.01 is monthly interval), origin at first month in period.

^b R' is the coefficient of multiple correlation corrected for the number of parameters.

^c ϵ is the standard error of estimate corrected for the number of parameters.

^d $y p'' dp/dt I t$ is the coefficient of part correlation. See M. J. B. Ezekiel, *Methods of Correlation Analysis*, New York, 1930, pp. 181-183. Defined generally, it is $r_{2121, 145} = x_{23} - e_{2, 345}$.

form and significant and indicate no change in direction from period to period. The net relation between quantity-taken and price is negative, between quantity-taken and the rate of change of price positive, between quantity-taken and an index of industrial production positive, and between demand and time generally positive. There is some change in the order of the coefficients for the war period, but the use of the logarithms of price would probably have reduced this difference. The coefficients of the indices of production are, unfortunately, not comparable. There is some indication, however, that the relations of price and the rate of change of price in the third period returned to their pre-war status.

3. The negative relation between quantity-taken and price can be described in other ways than by the regression coefficient. The part correlation coefficient¹¹ is, for our purpose, one of the best of these. This coefficient is defined as the correlation between one of the independent variables and the residuals of the dependent variable from its estimate by the other independent variables when these independent variables are given the weights of the general estimating equation. If, for example,

$$X_1 = a + b_{12.34}X_2 + b_{13.24}X_3 + b_{14.23}X_4,$$

the part correlation coefficient between X_2 and X_1 is the correlation between X_2 and $X_1 - b_{13.24}X_3 - b_{14.23}X_4$. In the present case this coefficient describes the relation between price and quantity-taken after the latter has been corrected, in the manner just described for the three factors, rate of change of price, industrial production, and time. An examination of Table II will show that in all cases this statistic ($_{12r_{345}}$) is negative and fairly high. It might be argued that while prices should not be corrected for industrial production or for the rate of change of price, they should be corrected for time. Experimenting with this modification in the first period gives a coefficient of -0.51 , which is nearly identical with the part correlation. No matter how it is computed, then, a significant net relation of a negative sort exists between quantity-taken and price.

5. COMPARISON OF THE GENERAL BUSINESS EQUATION AND THE EVANS-ROOS THEORY

The general business equation (6) gives, as might be expected, a better fit than the Evans-Roos equation (5). The coefficients of correlation are:

	Equation (5)	Equation (6)
1902-1915	0.68	0.81
1916-1920	0.67	0.88
1921-1930	0.75	0.92

¹¹ M. J. B. Ezekiel, *Methods of Correlation Analysis*, New York, 1930, pp. 181-183.

The results of fitting the two equations give interesting relationships. In each equation the relation between sales and the rate of change of price is positive and significant. Either demand pattern substantiates the bullish and bearish effect of rising and falling prices. But the outstanding feature is *the stability of the coefficients of the price term in both cases*. A comparison of the coefficients of price for the three periods indicate the following.

	Equation (5)	Equation (6)
1902-1915	-2.50 ± 1.00	-1.56 ± 0.31
1916-1920	-0.60 ± 0.25	-0.55 ± 0.08
1921-1930	-1.93 ± 1.09	-1.27 ± 0.23

In none of the three periods is the difference between the coefficients greater than the larger of the standard errors. This stability, of course, does not prove that quantity and price would have a relation of this order under all possible demand hypotheses, but it does indicate that relations obtained by fitting the two equations are more than a chance phenomenon.

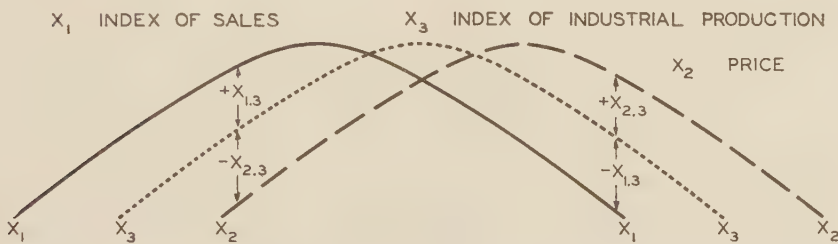


FIGURE 2

Thus, we may say that there now exist by the criteria of rationality, goodness of fit, reasonableness of sign, and size of the parameters, two fairly good demand equations for steel. It is possible to conclude that if either of two different sets of factors is given constant weights, the relation between price and quantity is negative and for both sets approximately of the same order.

The hypotheses made, however, are by no means the only possible ones. For second approximations, equations more complex and flexible might be needed, and when these more refined hypotheses have been stated and tested, the conclusion just drawn may need modification. This conclusion must also be qualified by the fact that the application of a multiple correlation technique gives only an average result for the period covered. Consider, for example, the relations of the cyclical movements of the series in the second experiment. The Index of Sales precedes price by approximately a quarter period. The Index of Industrial Production lags behind the Index of Sales by four or five months, and so will lie between the other two series. Figure 2 which is

an idealized picture of the typical situation, indicates that when the deviations of sales from the Index of Industrial Production are positive ($+x_{1.3}$), those of price are negative ($-x_{2.3}$), and vice versa. But partial correlation can be defined as $r_{12.3} = r_{x_{1.3}x_{2.3}}$ or as the correlation of the residuals of the first and second variables, each from their best estimate, which estimate is calculated from the remaining independent variables. The partial correlation between sales and price will, therefore, be negative, and the sign of the regression coefficient ($b_{12.3}$) negative also. This proves that the timing of the series is responsible, at least in part, for the negative price-quantity relation.

This, in turn, means that the general significance of the coefficients is limited. There is only one observation for each month, only one set of observations for each cyclical movement. The observations for each cyclical movement, furthermore, follow much the same pattern. This limited pattern of observations limits the generality of the coefficients we derive by fitting equations to these observations. Thus, there is no proof that the relations given by the coefficients may not be irreversible. That is, it is not certain, for example, whether a rise of prices early in the boom would have the same effect as one late in the boom. Such a contingency did not arise within the range of the observations. The fitted equations are a description of the average, though fairly uniform, relation of the series. It is quite possible that the action of other unanalyzed factors produces the observed concomitances. An attempt to analyze the possible effect of the position of the cyclical movement on the reaction of quantity-taken to price and to the movement of price will be a pertinent subject for further research.

Despite this qualification, this study is perhaps the first successful attempt to derive from the concrete facts of price behavior of this type of commodity an analysis which is at all plausible. It will, I believe, be the departure for a more rigorous and comprehensive analysis of quantity-price relations.

APPENDIX A

CONSTRUCTION OF INDEX OF SALES

The use of production data for demand study in the steel industry is unsatisfactory, because it involves the assumption that production of one month is approximately equal to the sales of that month. The fact is that the current production of steel products is not sold at current prices, for production is on order and prices are contract prices. There are, however, no sales data available. An index which will approximate sales must be constructed. The series used for this purpose are the monthly figures for the production of steel ingots,¹² and the monthly figures (quarterly prior to 1910), for the unfilled orders of the United

¹² *Iron Age*, Vol. 127, 1930, p. 145.

States Steel Corporation.¹³ The construction of the Index was as follows: taking the difference between the unfilled orders of the United States Steel Corporation at the end of successive months gave the monthly change in orders for that company. To estimate the change in orders for the entire industry, it was assumed that the orders of the United States Steel Corporation were in the same proportion to the orders of the entire industry as their production in the current year. If they produced 50% of the steel and had unfilled orders of 4 million tons, the entire industry was expected to have had orders of 8 million tons. If the total increase in the orders of the United States Steel Corporation (calculated) was 0.1 million tons, then that for the entire industry (estimated) was 0.2 million tons. The same results will be obtained if the changes in the unfilled orders of the United States Steel Corporation are weighted by the reciprocal of the proportion of the steel production enjoyed by that company during the current year. The figures for the total steel ingot production of that month were added to these weighted first differences. I have called this series the Index of Sales for steel. Its five-month moving average is plotted in Figure 1.

This index is an approximation to sales from the difference between the orders at the beginning and at the end of the month, plus the amount of the orders that have been filled through current production, and is an estimate of sales. The assumptions we have made are:

1. That the orders of the United States Steel Corporation are representative of the general situation in the industry. The proportion of the production by that company varies from 40 per cent to 60 per cent which precludes the possibility of greatly exaggerating any error in this assumption. The evidence for the assumption is chiefly of a negative sort. The one instance in which a marked difference between the Steel Corporation and the independents was recorded was in 1920, at which time the United States Steel Corporation had booked ahead of the other firms. There resulted for that year an unparalleled price situation in that the prices of the steel corporation were consistently lower than those of the independents.¹⁴ But this is not a frequent occurrence.

2. That there have been no marked changes in the manner in which the United States Steel Corporation records its orders. There is no certain information as to the validity of this assumption. It has been hinted that in recent years the company considers orders that will be filled over a period of time as unfilled orders only at the beginning of the month during which they will be filled. (This is the policy with respect to such orders as those of the American Can Company.)¹⁵ This might be the explanation of the reduction in the lead of sales over pro-

¹³ *Ibid.*, p. 148.

¹⁴ *Iron Age*, Vol. 107, 1921, p. 1.

¹⁵ News item in *Chicago Daily News*, February 9, 1932.

duction in the last few years. In the opinion of the trade,¹⁶ however, there has actually been an increase in hand-to-mouth buying. I conclude, therefore, that the changes in the recording of orders have not been sufficiently drastic to invalidate the Index although no adjustment can be made for this factor.

There is one more assumption that must be made. Granted that we have an index of orders, is it a measure of "quantity-taken"? Is it logically the proper dependent variable for a demand equation? We know that production is not the correct dependent variable and that this assumption must be made. But we should, if possible, check the assumption.

It is very difficult to do this. The United States Steel Corporation has never published a statement concerning the exact composition of their figures for unfilled orders. The following facts, however, are common knowledge of the trade:

1. There may be duplication of orders on the upswing and cancellations on the downswing, consequently the Index may exaggerate the cyclical swings in orders.

2. Orders may be placed with no definite specifications for their filling given and such specifications may be delayed indefinitely. A change in orders may, therefore, not represent purchases at the time of placement, but merely indicate probable future purchases. Such orders are made as a guarantee against a rise in prices.

3. A portion of the orders are genuine contracts for definite shapes, and represent actual current sales. So much is known, but the relative importance of the items is impossible to determine. A certain element of the Index of Sales, an element of unknown size, is purely speculative.

The real strength of the procedure in the study lies in this: while it is impossible to get a figure giving the "quantity-taken" of a Marshallian demand equation, it is possible to get an estimate of orders. The demand equation has, therefore, been adapted to this situation. In it there is a term expressing the rate of change of price. When the price starts to rise, the speculative type of order comes in to guarantee against future price rises. If there were no such elements in the steel market and orders were strictly "quantity-taken," it would be much less important to adopt a demand equation which takes this into consideration. As it is, there is a speculative element in the Index of Sales, and a speculative element in the demand equation. The results seem to be the best possible adaptation of method to data and facts.

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¹⁶ *Iron Age*, Vol. 119, 1927, p. 1380.

